PG – 856

I Semester M.Sc. Examination, January 2017
(R.N.S.) (2011 Onwards)
MATHEMATICS
M 103 : Topology – I

Time : 3 Hours   Max. Marks : 80

Instructions : *1) Answer **any five** questions choosing atleast **two** from **each** Part.*
*2) **All** questions carry **equal** marks.*

PART – A

1. a) Let $f : X \to Y$ be a one-one correspondence. If x is infinite then prove that y is infinite. **8**

   b) Prove that $\mathbb{N} \times \mathbb{N}$ is denumerable, where $\mathbb{N}$ is the set of natural numbers. **8**

2. a) Prove that the open interval (0, 1) of real numbers is a non-denumerable set. **8**

   b) State and prove Cantor's theorem. **8**

3. a) Define a metric on a set. If d is a metric on a set X, then prove that

   $$d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}, \text{ for all } x, y, \in X \text{ is a metric on } X.$$ **8**

   b) Show that subspace of a complete metric space is complete if and only if it is closed. **8**

4. a) State and prove contraction mapping theorem. **8**

   b) State and prove Cantor's intersection theorem. **8**

PART – B

5. a) Let $(X, \mathcal{T})$ be a topological space. If A is any subset of X and d (A) is its derived set then prove that $A \cup d(A)$ is closed. **8**

   b) With usual notations prove the following : **8**

   i) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

   ii) $(A \cap B)^{\circ} = A^{\circ} \cap B^{\circ}$.

P.T.O.

6. a) Show that a function $f : X \to Y$ is continuous if and only if inverse images of open sets are open. **7**

   b) Given a function $f : X \to Y$, prove that the following are equivalent : **9**

      i) f is continuous

      ii) B is closed in Y implies f^{-1} (B) is closed in X

      iii) $f(\overline{A}) \subseteq \overline{f(A)}$ for all $A \subseteq X$.

7. a) State and prove pasting lemma. **5**

   b) Define a connected set. Show that the closure of a connected set is connected. **5**

   c) If $\{C_\lambda\}$ is a family of connected subsets of $(X, \mathcal{T})$ such that $\bigcap_\lambda C_\lambda \neq \phi$ then prove that $\bigcup_\lambda C_\lambda$ is connected. **6**

8. a) Define component of a topological space. Show that components are closed. **5**

   b) Prove that a path connected set is always connected. **5**

   c) Show that the image of a locally connected set under continuous open map is locally connected. **6**